Logical groundwork

Principle 1

There are geometrical entities, geometrical qualities and geometrical places.  
Principle 2

There are Identities made from entities and qualities. Entities and qualities that make an identity are called identifiors.

Principle 3

A single place places a geometrical entity. It is written

Principle 4

An identity identifies kinds of Geometrical qualities and entities. A kind of qualities or entities is identified by a list of kinds of identifiors if and only if such a list could construct a particular entity of the kind in any place, or a particular quality of the kind. It is written

Principle 5

Particular Entities and qualities which has the same identity are called identical.Identical entities and qualities are equivalent with each other for the sake of being identifiors.

means is identical to

means is identical and in the same place as

Principle 6  
if a basic identity of entity include n entities of the kind , and if there is an identity of q that include identifiors of less, a particular is identified by and particular s with carefull attantion to order.

but

Principle 7

There are also things that are merely devices, they are not exactly part of the construction, but they make it easier for us to talk about the entities and the qualities. Those cannot be identifiors.

Place

*Place* is self-evident

Point

**Define**: A *Point* is an entity placed by a single place and defined by a blank identity.

Therefore, all points are identical

Direction

*Direction* is a self-evident quality.

**Define:** direction is the mode of difference in place between two points.

1. For any 2 point there is a corresponding direction

**Clarification:** there is no direction corresponding to a point and itself

1. Direction is anticommutative
2. It is transitive

And it is weakly backwardly transitive.

**Define:** An *Orientation* is a device to call 2 opposite directions.

**Define:** A *Directional* *Value* is a real number value given to a place relative to a certain direction.

It is a device.  
such that

Distance

**Define:** A *Distance* is the amount of difference in place between 2 points along their orientation.

Distance is a quality

Such that

**Clarification:** Distances could have any real, non-negative number value.

**Postulate 1:** [[1]](#footnote-1)  
**explanation:** for any given place direction and distance, exist a place such that the direction between the two places' points equal the given direction, and the distance between this points equal the given distance.

Vector

**Define:** A *Vector* is the difference in directionsl value, in a certain direction.

It is a device

It could be identified by a pair of points and a direction

By a pair of points alone

Or by a distnce and a dirction

Such that

**Lema 1**:

**Proposition 1: Explanation:** Vectors that has the same orientation obey the regular laws of arithmetics.

It's easy to demonstrate that

Length

**Define:** A*Path* is an entity and a well-defined continuous sequence of points.

path always has a pair of points in the identity. The one being the first object, the other being the last.

**i.e.** All paths are finite.

**Define:** A*Length* is a quality, and a sum of the distances of all pairs of Consecutive points along a path

LINES

**Define:** A *Line* is an entity and a set of points, identified by two points

such that all pairs of points, have common direction with the original pair

and their directional value by that direction is bound by the directional value of those two points.

It is placed by its first point

**Reverse:** if a point is bound by the two points of the line and have their direction with a point belonging to the line it belongs to the line

**Define:** a *Straight Path* is a path defined by a pair of points and their direction

it starts with the first of the pair, for any point belonging to the path the next point is the closest such that the direction between them is the direction of the original pair. When the sequence reaches the second point in the pair it stops.

the terms of a path we'll notaten a certain direction.

**Proposition 2**: **Explanation:** the line and the set of a straight path identified by the same pair of points in the same order are equivalent

Therefore, there is one unique length for any line

**Proposition 3**:  
**Explanation:** the length of a line equal the distance between the two point defining it.

**Proposition 4:**   
**Explanation:** a line is equal to a a line with the opposite direction.

**Proposition 5:** a line could also be identified by its length (which is the distance of its points) and orientation.

Cross

**Define** a *cross* is a point that belong to 2 lines.

**Reverse:**  line that has a cross with each other are crossing lines.

**Define:** *parallel lines* are lines that have the same direction while having a different direction between points on the first and points on the other

*Some Propositions about crossage of lines*

**Proposition 6: Explanation:** parallel lines don't cross.

**Proposition 7A:**

**7B:**  
**Explanation:** if lines have the same direction and this direction is also the direction between all points belonging to the one and all points belonging to the other, some range of points could belong to both.

**Proposition 8:**   
**Explanation:** if lines have different orientation, one and at most one cross is possible

**Proposition 9**: **Explanation:** the distance between first ends of equal parts of parallel lines, and the distance between the other ends, are themselves equal.

Angle

**Define:** angle is the difference in direction.

Angle is a quality.

Angles are anticomotative

and transitive

**Note**: directions do not act like real numbers hence

Angles themselves could have real number value.

**Identification:** angles could also be identified by points such that:

**Postulate 2:** the angle between any 2 opposite directions is the same

If they are all equal the same, it follows that there is such constant value that they all equal to. We shall denote this constant π.

**Proposition 9**:   
**explanation:** the angle between any direction and itself is 2π

**Proposition 10:**

From this follows that for any two directions, there is infinite number of angles, some are positive, and some are negative[[2]](#footnote-2), as k could be positive or negative.

**Define:** angular direction, is the mode of difference in direction.

Ang. Direction is a quality.

For any two directions there are two angular directions.

For convenience we shall term the one ang. direction cw and the other ccw. And we shall make an arbitrate choice that ccw is the positive direction unless said otherwise. So ccw will be further notated + and cw -.

The ang. directions could be changed to the other by flipping the order of the directions (anticomotativity), changing the sign, or multiplying by -1.

**Define:** a reminder of an angle is a value between 2π and 0 that equal to a value of the angle.

Such that

We are using the ccw is positive standard, the reminder is always ccw.

Reminder is a tool.

**Proposition 11: Explantion:** exist only one reminder for each angle.

Comparison of angles is defined to the same k such that

Hence

**Lemma 2:**

and

**Proposition 12:**

**Proposition 13:**

**Lemmma 3:**

And the same with flipped comperators.

**Define:** plane is a set of places defined by three points

Such that

In this treatise, we will assume that all the points are in the same plane. If a treatise about space geometry shall be written everything said about more than 3 points will be again exemined.

**\*\***

**Proposition 12:**

**Postulate 3:**

**Define:** *Perpendicular Directions*are direction that construct an angle of

**Proposition 13:**

**Preposition ?:**

**Proposition 14:**

**Proposition 15**:

**Proposition 16:**

Area

**Define**: A *Height* is a line bound by a point and a line which have a perpendicular direction to the line

**Define:** an *internal direction* for an angle is a direction that is included in the angle

Circle

**Define:** A circle is a set of points identified by a distance and placed by a point

Such that

is called the center of the circle. is called the radius of the circle.

**Propositions:** about the crossing of lines and circles and circles and circles

**Propositions 9:** the line from a point on the circle to the point on the other side that include the centre of the circle divide circle by two.

The length that is defined by both of the paths along the circle is the same and amount to:

If p\_o∈l\_a where po is a centre of a circle O\_b than l\_a and O\_b shere 0,1 or 2 points.   
if l\_a<r\_o where r\_o is the distance of the O\_b. The circle and the line shere 0 points.  
if l\_a≥r\_o they shere 0 or 1 points.  
if l\_a≥2∙r\_o they shere 2 points.

|  |  |  |
| --- | --- | --- |
| ≥r | <r |  |
| 1 | 0 | <r |
| 2 | 1 | ≥r |

**Define:** an arc is a segment of a circle defined by a distance and three points

∩(r,p\_∩ p\_a p\_b)

such that p\_c∈∩(r\_∩ p\_∩ p\_a p\_b )iff r(p\_c,p\_(p\_∩ ) )=r\_∩  
p\_∩ is called the centre of the arc

and path(∩(r\_∩ p\_∩ p\_a p\_b ) )=path(cd(+),p\_a p\_b)

If p\_∩∈l\_a where p\_∩ is a centre of an arc ∩\_b than l\_a and ∩\_b shere 0 or 1 points.  
if l\_a<r\_o where r\_o is the distance of ∩\_b. The arc and the line share 0 points.  
if l\_a≥r\_o they shere 0 or 1 points.

**Define:** an angle of an arc is defined  
ang(∩(r\_∩ p,p,p) )=lng(∩)/r\_∩

Angle amount the change in direction such that  
for∩(r\_∩ p\_∩ p\_a p\_b ):|dir(p\_∩ p\_a )-dir(p\_∩ p\_b ) |=ang(∩(r\_∩ p\_∩ p\_a p\_b ))

To finish a new way to define the arc is

∩(r,p\_∩ p\_a p\_b )=∩(r,ang)[p\_∩]

an arc could be constructed on two lines that share an end point by choosing a distance smaller than any of the lines

∀ l(p\_a p\_b )l(p\_a p\_c ) r\_d∃∩(r\_d p\_a p\_e p\_f ):r\_d< l(p\_a p\_b ),l(p\_a p\_c );〖 p〗\_e∈l(p\_a p\_b ) ;  
p\_f∈l(p\_a p\_c );r(p\_a p\_e )=r(p\_a p\_f )=r\_∩

Such that an angle called be defined for a pair of lines or for three points

if p\_c∈l\_a,l\_b than ang(∩(l\_a l\_b,r\_d ) )=ang(l\_a l\_b )  
∀p\_d∈l\_a,p\_e∈l\_b

Angles is a change in direction

in a circle there is as much points as directions, so if we sum the directions, we will also sum the distances In a path from a point to itself along the circle (provided we demonstrated a circle end in the point in which it started)

***להגדיר את הקו באופן הקודם, ואז להוכיח שהוא רציף לכן דרך לכן יש לו אורך***

1. This actually should be part of the definition of the place, but it requires the distance which is a construction of the place so to be completely hierarchal this is a postulate. [↑](#footnote-ref-1)
2. Those positivity and negativity are of course relative to a arbitrary choice. It is only important that the relation between them is as that of positive and negative. [↑](#footnote-ref-2)